

Lesson 3-1: Properties of Parallel Lines

Parallel lines

Two or more lines are parallel *iff* they are coplanar and do not intersect. As you read that statement, perhaps several things ran through your mind:

1. That statement is a biconditional.
2. That statement is a definition (and if you examine it, it is a good definition because it satisfies the three rules for a good definition).
3. "I knew that." (...did you? Did you remember the definition includes the very important qualifier that the lines are coplanar?)

Today, we are going to continue our journey exploring the world of geometry by playing around with parallel lines. Our goal is to discover (and prove) some interesting properties of parallel lines.

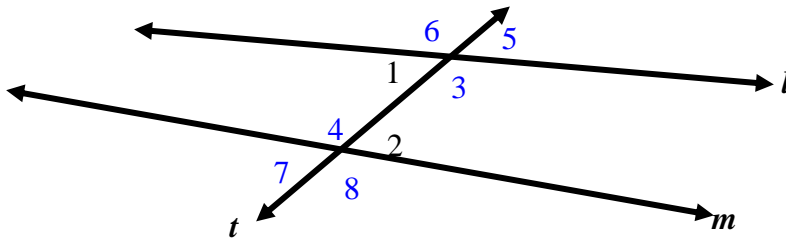
Angles, angles, angles...

Last lesson, we talked about some very important angle pairs: vertical, adjacent, complementary and supplementary angles. Well, we're not done with angles. If you think about it, angles are what describe how coplanar lines relate to each other.

But first, we need some definitions so we can understand what we're talking about.

Transversal line

A line that intersects two coplanar lines at two distinct points is called a **transversal**. The transversal will create eight angles as the following diagram shows. The angles do not need to be numbered in any particular order. Line t transverses lines l and m :



The angles that lie between the traversed lines are called interior angles: $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$ are all interior angles.

As you may guess $\angle 5$, $\angle 6$, $\angle 7$ and $\angle 8$ are all exterior angles.

These eight angles form three different types of angle pairs.

Alternate interior angles

Alternate interior angles are interior angles that lie on opposite sides of the transversal. In the diagram above $\angle 1$ and $\angle 2$ are alternate interior angles.

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Same-side interior angles

Same-side interior angles are interior angles that line on the same-side of the transversal. In the diagram above $\angle 3$ and $\angle 2$ are same-side interior angles.

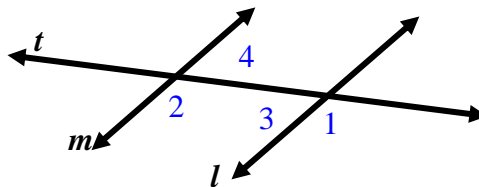
Corresponding angles

In the diagram above $\angle 1$ and $\angle 7$ are corresponding angles. There are three other pairs of corresponding angles; can you spot them? They are 4 & 6, 2 & 5, and 3 & 8.

Classifying angles

A very common task you will need to accomplish is to classify the angles in a diagram. Try your hand with the following:

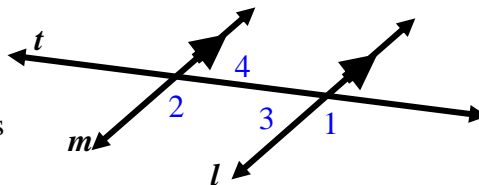
- a) Classify $\angle 1$ and $\angle 2$.
- b) Classify $\angle 2$ and $\angle 3$.
- c) Classify $\angle 3$ and $\angle 4$.



The angles of the first set are corresponding angles. The angles of the second set are same-side interior angles. The angles of the third set are alternate interior angles.

A special case

If a transversal intersects two parallel lines, we can say a bit more about the relationships among the angles formed. Let's explore this a bit. If you consider the diagram above, lines l and m are parallel. In a diagram, we use special markings (arrow heads in the middle of the lines) to indicate lines are parallel. Here is that diagram repeated with the parallel line markings included:



This diagram clearly indicates that lines l and m are parallel.

We have a special symbol for parallel: we can say it this way: $l \parallel m$.

Postulate 3-1 Corresponding Angles Postulate

If two parallel lines are cut by a transversal, then corresponding angles are congruent.

Can we prove this postulate?

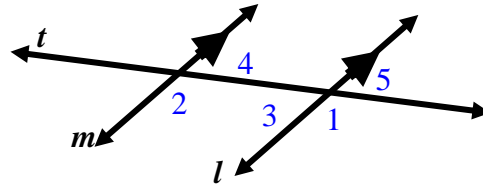
Remember, a postulate is something we simply accept as truth. It is one of the fundamental building blocks of geometry. Now you may be wondering, how in the world do I know this is really true??? Well, to be honest we don't. We are simply making a rule that we are going to build upon. We could certainly change this rule if we wanted. However, if we did that, then all of our following discoveries would be different wouldn't they? In fact, Euclid stated this postulate (or one very similar) and mathematicians spent many centuries trying to prove it as a theorem. They couldn't. Finally, in the 19th

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century (remember, Euclid lived way back around 300BC!) mathematicians decided they'd see what would happen if they changed this rule. When they did so, they discovered some very interesting and very different geometries. For now, or at least until you are in college and take a non-Euclidean geometry course, let's just leave this postulate as is.

Exploring a bit...

Here is our diagram again:



We now know that corresponding angles are congruent: $\angle 1 \cong \angle 2$.

Given this information, what can we say about the relationship between $\angle 3$ and $\angle 4$? Form a conjecture. To help you get started, classify them. They are alternate interior angles.

If you are having a hard time, fill in the blank on this conditional:

If two parallel lines are cut by a transversal, then alternate interior angles are _____.

If your conjecture is they are congruent, that is an excellent conjecture. Let's see if we can prove this conjecture.

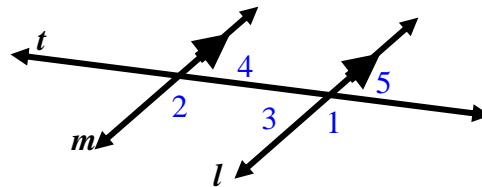
Proving alternate interior angles of traversed parallel lines are congruent

We need a plan. First, what information do we have to start with? Well, we know that $l \parallel m$. We want to prove that $\angle 3 \cong \angle 4$. Our plan...start with $\angle 3$ and see if there is another angle that we can do something with. What about $\angle 5$? Can we say something definite about how $\angle 3$ and $\angle 5$ relate? Sure! They are vertical angles, and we know by the Theorem 2-1 that vertical angles are congruent. Now look at $\angle 5$ and $\angle 4$; they are corresponding angles! By Postulate 3-1 (above) we know that corresponding angles are congruent. We're set! (Actually this is the proof in paragraph form...)

The proof

Given: $l \parallel m$

Prove: $\angle 3 \cong \angle 4$



Proof: $l \parallel m$

$\angle 3 \cong \angle 5$

$\angle 5 \cong \angle 4$

$\angle 3 \cong \angle 4$

Q.E.D.

Given

Vertical angles are congruent (Theorem 2-1)

Corresponding angles are congruent (Postulate 3-1)

Transitive property of congruence

We now have developed another theorem.

Theorem 3-1 Alternate Interior Angles Theorem

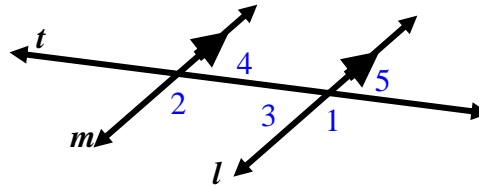
If a transversal intersects two parallel lines, then alternate interior angles are congruent.

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Exploring some more...

Consider this diagram. Line $l \parallel m$.

This time try to form a conjecture about $\angle 2$ and $\angle 3$. Start by classifying them. They are same-side interior angles.



Did you come up with something like this?

If two parallel lines are cut by a transversal, then same-side interior angles are supplementary.

Again, we need a plan. We want to prove that $\angle 2$ and $\angle 3$ are supplementary, or in other words that $m\angle 2 + m\angle 3 = 180$. This is our end point. What we need to do is get things set up as an algebra equation and boil it down to $m\angle 2 + m\angle 3 = 180$. Question is how do we get started?

Are there some other angle relationships we can make statements about? What about $\angle 1$ and $\angle 3$? $\angle 1$ and $\angle 3$ are supplementary angles. Excellent! We can make an algebra statement about that.

See any other possibilities? What about $\angle 1$ and $\angle 2$? They are corresponding angles and postulate 3-1 says they are congruent. Let's run with it...

The proof

Given: $l \parallel m$

$\angle 1$ & $\angle 3$ are supplementary (and therefore sum of their measures is 180)

$\angle 1$ & $\angle 2$ are corresponding angles (and therefore are congruent)

Prove: $\angle 2$ & $\angle 3$ are supplementary

Proof: $l \parallel m$

$$m\angle 1 + m\angle 3 = 180$$

$$m\angle 1 = m\angle 2$$

$$m\angle 2 + m\angle 3 = 180$$

$\angle 2$ & $\angle 3$ are supplementary Defn supplementary angles

Q.E.D.

Given

\angle Addition Postulate (supplementary angles)

Corresponding angles are congruent (Postulate 3-1)

Substitution Property of Equality

Defn supplementary angles

Theorem 3-2 Same-Side Interior Angles Theorem

If a transversal intersects two parallel lines, then same-side interior angles are supplementary.

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Finding measures of angles

Using Postulate 3-1 and Theorems 3-1 & 3-2, we can discover a lot about angles formed by parallel lines.

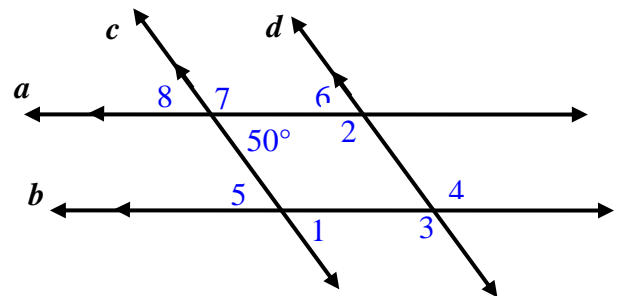
Example – Pg 118 Example 4 & Check Understanding 4

Find $m\angle 1$ and then $m\angle 2$. Which theorem or postulate justifies each?

Since $a \parallel b$, $m\angle 1 = 50$; Corresponding \angle 's Post.

Since $c \parallel d$, $m\angle 2 = 130$; Same-Side Int \angle 's Post.

- $m\angle 3 = 130$; Corr. \angle 's are congruent
- $m\angle 4 = 130$; Vert. \angle 's are congruent
- $m\angle 5 = 50$; Alt Int. or Vert. \angle 's are congruent
- $m\angle 6 = 50$; Alt Int. \angle 's are congruent
- $m\angle 7 = 130$; Same-Side Int. \angle 's are congruent (or suppl)
- $m\angle 8 = 50$; Corr. or Vert. \angle 's are congruent



Example – Pg 118 Example 5

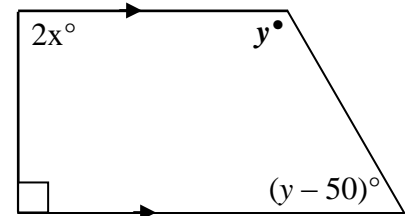
Work through this example. One thing to note is the use of the Angle Addition Postulate with three angles. The Angle Addition Postulate is not restricted to just two angles. If you divide an angle into five different parts, the Angle Addition Postulate says that the measure of the full angle is the sum of measures of each adjacent angle.

Example – Pg 118 Check Understanding 5

Find the values of x and y . Then find the measures of the angles.

Since:

- the top and bottom lines are parallel
- they are transversed by the vertical left line
- the bottom left angle is a right angle (measure = 90°)
- the $2x^\circ$ angle and the right angle are supplementary (same-side int \angle 's theorem)



...we can therefore conclude that:

- $2x + 90 = 180$ \angle Add Post
- $2x = 90$ subtr POE
- and $x = 45$ div POE

Further:

- the top and bottom lines are parallel
- they are transversed by the slanted right line
- the y° angle and the $(y - 50)^\circ$ angle are supplementary (same-side int \angle 's theorem)

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...we can therefore conclude that:

1. $y + (y - 50) = 180$ \angle Add Post
2. $2y - 50 = 180$ Simplify
3. $2y = 230$ Add POE
4. $y = 115$ Div POE

The angle measures are: 90, 115, 90, 65

Assign homework

p. 118 #1-9, 11-25, 30, 32, 33, 39, 40, 45